Geometry formulas & conditions

For revision. Not the official testing reference sheet or permission to bring notes into the exam.

Angle totals

$$a + b = 90^\circ \text{ or } 180^\circ$$

Complementary or supplementary, respectively; identify the relationship first.

Triangle angle sum

$$A + B + C = 180^\circ$$

Interior angles of a Euclidean triangle.

Triangle inequality

$$|a - b| < c < a + b$$

All three side lengths are positive; endpoints are excluded.

Triangle area

$$K = \frac{1}{2}bh$$

The height is perpendicular to the chosen base.

Rectangle and parallelogram

$$K = lw, \qquad K = bh$$

Use a perpendicular height for a parallelogram.

Trapezoid

$$K = \frac{1}{2}(b_1 + b_2)h$$

The bases are parallel.

Polygon angles

$$S = (n - 2)180^\circ, \qquad E_{\text{regular}} = \frac{360^\circ}{n}$$

The equal exterior-angle formula requires a regular polygon.

Circle area and circumference

$$K = \pi r^2, \qquad C = 2\pi r$$

Use radius, not diameter.

Arc length

$$s = \frac{\theta}{360^\circ}2\pi r$$

Here θ is in degrees.

Sector area

$$K = \frac{\theta}{360^\circ}\pi r^2$$

A sector includes two radii and an arc.

Distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Coordinate differences are the legs of a right triangle.

Midpoint

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Average each coordinate independently.

Partition point

$$P = A + \frac{m}{m + n}(B - A)$$

Internal ratio $AP : PB = m : n$.

Slope and line

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad y - y_1 = m(x - x_1)$$

The slope form excludes vertical lines.

Circle equation

$$(x - h)^2 + (y - k)^2 = r^2$$

Center (h, k), positive radius r.

Transformations, solids & right triangles

Translation and dilation

$$(x, y) \mapsto (x + a, y + b), \quad (x, y) \mapsto (kx, ky)$$

The dilation shown is centered at the origin.

Quarter-turn counterclockwise

$$(x, y) \mapsto (-y, x)$$

Rotation of 90° about the origin.

Reflection in axes

$$(x, y) \mapsto (x, -y), \quad (x, y) \mapsto (-x, y)$$

Across the x-axis and y-axis, respectively.

Scaling

$$L \mapsto kL, \quad K \mapsto k^2K, \quad V \mapsto k^3V$$

Uniform positive similarity scale factor k.

Prism and cylinder volume

$$V = Bh, \qquad V_{\text{cylinder}} = \pi r^2 h$$

Use perpendicular height.

Pyramid and cone volume

$$V = \frac{1}{3}Bh, \qquad V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

Use perpendicular height rather than slant height.

Sphere

$$SA = 4\pi r^2, \qquad V = \frac{4}{3}\pi r^3$$

A hemisphere's total surface may also include its base.

Right-prism surface area

$$SA = 2B + Ph$$

P is the perimeter of the base; both bases are exposed.

Closed-cylinder surface area

$$SA = 2\pi r^2 + 2\pi rh$$

Adjust if a base is open or hidden.

Right-cone surface area

$$SA = \pi r\ell + \pi r^2, \qquad \ell^2 = r^2 + h^2$$

The lateral area uses slant height ℓ.

Pythagorean theorem

$$a^2 + b^2 = c^2$$

Only for right triangles; c is the hypotenuse.

Special right triangles

$$1 : 1 : \sqrt{2}\,, \qquad 1 : \sqrt{3} : 2$$

45°–45°–90° and 30°–60°–90°, ordered by opposite angle.

Right-triangle ratios

$$\sin\theta = \frac{O}{H}, \quad \cos\theta = \frac{A}{H}, \quad \tan\theta = \frac{O}{A}$$

O opposite, A adjacent leg, H hypotenuse, for an acute angle.

Circle segment products

$$ab = cd, \qquad e_1 w_1 = e_2 w_2, \qquad t^2 = ew$$

Intersecting chords; external secants; tangent and secant, respectively.

Population density

$$D = \frac{\text{population}}{\text{area}}$$

Report people per unit area.

Theorem checks: Congruence: SSS, SAS, ASA, AAS, right-triangle HL. Similarity: AA, proportional SAS, proportional SSS. Parallel lines are needed for corresponding/alternate angle equality. An inscribed angle is half its intercepted arc; cyclic opposite angles are supplementary. Parallelogram diagonals bisect; rectangle diagonals are equal; rhombus diagonals are perpendicular.